

## The universal Kähler modulus in warped compactifications

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JHEP01(2009)036

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## The universal Kähler modulus in warped compactifications

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**ABSTRACT:** We construct the effective theory of the universal Kähler modulus in warped compactifications using the Hamiltonian formulation of general relativity. The spacetime dependent 10d solution is constructed at the linear level for both the volume modulus and its axionic partner, and nontrivial cancellations of warping effects are found in the dimensional reduction. Our main result is that the Kähler potential is not corrected by warping, up to an overall shift in the background value of the volume modulus. We extend the analysis beyond the linearized approximation by computing the fully backreacted 10d metric corresponding to a finite volume modulus fluctuation. Also, we discuss the behavior of the modulus in strongly warped regions and show that there are no mixings with light Kaluza-Klein modes. These results are important for the phenomenology and cosmology of flux compactifications.

**KEYWORDS:** Flux compactifications, Supersymmetric Effective Theories.

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**1. Introduction**

String backgrounds contain various light fields, such as metric zero modes and spacetime tensors. Determining their 4d dynamics, i.e., dimensional reduction, is essentially a two-step procedure. It requires first finding the correct 10d (in the case of string theory) fluctuation corresponding to the 4d field. Then the 4d action is computed by substituting

this fluctuation ansatz into the 10d action. In some cases, the first step is simple, which can lead to confusion in more complicated backgrounds. In this paper, we continue along the lines of [1–3] to advance the proper treatment of dimensional reduction in conformally Calabi-Yau warped compactifications of type IIB string theory [4–7].

The moduli space by dimensional reduction is well understood for Calabi-Yau (CY) compactifications, mainly because  $\mathcal{N} = 2$  supersymmetry determines the action in terms of a single prepotential and the zero modes are associated to harmonic forms of the CY [8]. However, Kaluza-Klein reduction in flux compactifications with  $\mathcal{N} = 1$  supersymmetry is much less understood. From a field theory point of view, finding the Kähler potential is a complicated task because it is no longer protected by holomorphicity. From a geometrical point of view, breaking  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  corresponds to having a warped background, with the warp factor sourced by branes, orientifold planes, and supergravity flux. Dimensionally reducing on these backgrounds is subtle, since the profiles of the zero and higher Kaluza-Klein (KK) modes are nontrivial [9–14].

Understanding the dynamics in these cases is an important task, because flux compactifications have many of the necessary ingredients to produce realistic models of phenomenology and cosmology [4–7]. Understanding that phenomenology therefore requires a knowledge of the proper dimensional reduction. For example, the 10d wavefunction controls the interactions of supergravity moduli with brane fields, which may represent the Standard Model, as in [15, 16]. Additionally, supergravity KK modes are dark matter candidates in some models; understanding their full 10d structure is important in determining whether their annihilation and decay rates are sufficiently slow [17–21].

In this paper, we study dimensional reduction in the compactifications of [4–7]. These are in some senses simpler than other flux compactifications, both because the internal space is conformally CY and because they have a well-defined supergravity limit. Progress in this direction has been made in [14, 2], especially with respect to volume-preserving fluctuations of the internal space. Recently, in [1], a formalism for computing kinetic terms in general warped backgrounds was developed which makes the physical interpretation of the computations manifest. Since this formalism does not rely on supersymmetry, it applies to conformally CY flux compactifications with flux that breaks supersymmetry as well (see [22] for some discussion of supersymmetry breaking in these backgrounds).

Our goal in the present work is to elucidate the dynamics of the universal Kähler modulus, applying the Hamiltonian-based method developed in [1] (see also [23, 14, 24] for progress towards a 10d description). This mode arises in any string background with a geometric interpretation, but its kinetic term has not yet been fully understood in the case of general warping. A particularly important question is how warping effects correct the kinetic terms and Kähler potential (for  $\mathcal{N} = 1$  theories). We will find that the Kähler potential is in fact not corrected by warping, up to an additive shift in the background value of the modulus. This is a rather surprising outcome, because the 10d solution constructed from the Hamiltonian method is quite different from the unwarped fluctuation. However, the needed shift in the modulus would affect nonperturbative superpotentials or higher-derivative corrections that break the no-scale structure of the classical background [25–28].

Most of the methods developed so far apply to moduli dynamics in the linearized

approximation, namely when the fluctuations around the vacuum expectation values are infinitesimal. This is certainly enough if one is interested in the Kähler potential and mass spectrum of the theory. However, understanding other effects, particularly in cosmology, beyond the 4d effective field theory requires going beyond the linearized level. For this reason, we also extend our approach to the case of finite spacetime dependent fluctuations of the volume modulus. This not only should serve to eliminate remaining confusion about the relation between the 10d and 4d theories, but it is also a significant first step in developing cosmological solutions of compactified 10d supergravity. Such solutions would demonstrate what signatures higher-dimensional or string physics could be generated, for example, by inflation in string theory [29].

Throughout, we restrict to conformally CY flux compactifications, but our method could be applied to more general  $\mathcal{N} = 1$  and nonsupersymmetric backgrounds as well.

### 1.1 Beyond the Calabi-Yau case

Before starting our analysis, it is instructive to review the simpler case of a Calabi-Yau compactification without warping. We follow the discussion of [7] for IIB CY compactifications. The universal volume modulus corresponds to a simple rescaling

$$\tilde{g}_{ij} \rightarrow e^{2u} \tilde{g}_{ij} \tag{1.1}$$

of the internal CY metric  $\tilde{g}_{ij}$ . The time-dependent metric fluctuation is, at linear order,

$$ds^2 = e^{-6u(x)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2u(x)} \tilde{g}_{ij}(y) dy^i dy^j, \tag{1.2}$$

where the 4d Weyl factor  $e^{-6u(x)}$  is needed to decouple the modulus from the graviton. This 4d rescaling defines the 4d Einstein frame and gives the Einstein-Hilbert action for the metric in 4d. The Einstein equations then reduce to the desired  $\square u = 0$  for the modulus. The 4-form field contributes an axion

$$C_4 = \frac{1}{2} a(x) \tilde{J} \wedge \tilde{J} + \dots \tag{1.3}$$

( $\tilde{J}$  is the fixed Kähler form associated with the fixed CY metric  $\tilde{g}_{ij}$ ), which pairs with the volume modulus into the complex field  $\rho = a + ie^{4u}$ . Performing the dimensional reduction, one finds

$$K = -3 \log(-i(\rho - \bar{\rho})) . \tag{1.4}$$

Backreaction from fluxes and branes (of the BPS type discussed in [7]) introduces warping to the background,

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{ij} dy^i dy^j . \tag{1.5}$$

One could then try different ways of identifying the universal volume modulus. The simplest possibility would be to consider the same dependence as in (1.2), even in the presence of warping [30]:

$$ds^2 = e^{2A(y)} e^{-6u(x)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} e^{2u(x)} \tilde{g}_{ij}(y) dy^i dy^j . \tag{1.6}$$

This proposal does not work for a couple of reasons. Under a spacetime-independent rescaling  $\tilde{g}_{ij} \rightarrow e^{2u}\tilde{g}_{ij}$ , the warp factor acquires a dependence on  $u$

$$e^{-2A} \rightarrow e^{-2u} e^{-2A} \tag{1.7}$$

in such a way that the full internal metric  $e^{-2A}\tilde{g}_{ij}$  is actually invariant under the rescaling. Therefore, the simple rescaling of the CY metric becomes a gauge redundancy which may be set to zero by a 4d Weyl transformation. At a more technical level, eq. (1.6) cannot solve the 10d Einstein equations, so it does not give a consistent time-dependent fluctuation.

Another possibility is suggested by the fact that the warp factor is only determined up to an overall shift,

$$e^{-4A(y)} \rightarrow e^{-4A(y)} + c . \tag{1.8}$$

The volume of the compact space scales as  $V \sim c^{3/2}$ , so it would be natural to identify this flat direction as the warped version of the universal volume modulus. One could then promote  $c$  to a spacetime field  $c(x)$  by considering the metric fluctuation [23, 24, 14]

$$ds^2 = [c(x) + e^{-4A_0}]^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + [c(x) + e^{-4A_0}]^{1/2} \tilde{g}_{ij} dy^i dy^j \tag{1.9}$$

and performing the dimensional reduction. However, this proposal does not solve the linearized equations of motion<sup>1</sup> either; additional components of the metric are required to satisfy all the components of the 10d Einstein equation [14]. Dimensional reduction on backgrounds for which the 10d equations of motion are not satisfied in general does not lead to good low energy effective theories, and can result in ambiguities, as noticed in previous studies [11–13, 23, 24].

Summarizing, the dynamics of the universal Kähler modulus are not understood beyond the CY case, and a more systematic approach is needed. In this paper we will use the method proposed in [1] to find the wavefunction for the volume modulus and its axionic partner in the presence of warping. This approach can also be extended to more general  $\mathcal{N} = 1$  or nonsupersymmetric backgrounds.

## 2. Review of the Hamiltonian approach

The main obstacle in computing the 4d action is the appearance of “compensating” fields [14, 31]. These arise in any system with gauge redundancies and time-dependent fields. In a Lagrangian formulation their role is not manifest. If they are not taken into account properly, the low energy effective action is not invariant under 6d diffeomorphisms, making the description inconsistent. In [1] it was shown that a simple way of deriving the correct gauge invariant action is in the Hamiltonian framework. The compensators are then identified as Lagrange multipliers, and their dynamical role becomes manifest. For completeness, in this section we summarize the results of [1].

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<sup>1</sup>Except for special choices of  $c(x)$  which appear to lead to instabilities [23, 24].

## 2.1 Gauge invariant fluctuations

Consider a warped 10d background preserving 4d maximal symmetry,

$$ds^2 = e^{2A(y; u)} \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + g_{ij}(y; u) dy^i dy^j, \quad (2.1)$$

which depends on metric zero modes  $u^I$  (which do not mix with the 4d metric at linear order). The kinetic terms for  $u^I$  are obtained by promoting the modes to spacetime dependent fields  $u^I(x)$ . However, the new metric (2.1) with spacetime dependent  $u^I(x)$  is generically (for non-trivial warp factor) no longer a solution of the 10d Einstein equations. In particular, the mixed component of the Ricci tensor  $R_{\mu i}$  acquires a term proportional to  $\partial_\mu u^I$  and becomes nonzero. Ansätze of this form are therefore not viable starting points for KK dimensional reductions.

This problem is solved at linear order in velocities by including compensators  $\eta_{Ij}$ ,

$$ds^2 = e^{2A(y; u)} \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + 2\eta_{Ij}(y) \partial_\mu u^I dx^\mu dy^j + g_{ij}(y; u) dy^i dy^j. \quad (2.2)$$

The spacetime dependent metric fluctuation (2.2) and the 4d kinetic term are then obtained by solving the 10d Hamiltonian equations of the warped background. We refer the reader to [32] for the formulation of general relativity in canonical variables.

The formalism is based in computing the canonical momentum  $\pi^{MN}$ , which may be seen to be equal to

$$(g^{tt} h)^{-1/2} \pi_{MN} = \left( \dot{h}_{MN} - D_M \eta_N - D_N \eta_M \right) - h_{MN} h^{PQ} \left( \dot{h}_{PQ} - D_P \eta_Q - D_Q \eta_P \right) \quad (2.3)$$

where  $h_{MN}$  is the 9d space-like metric with components  $(g_{\mu\nu}, g_{ij})$  for  $\mu, \nu \neq 0$ .  $D_M$  is the covariant derivative constructed from  $h_{MN}$ , and  $\eta^N = \dot{u}^I \eta_I^N$ . Then the Hamiltonian density becomes

$$\mathcal{H}_G = \sqrt{-g_D} \left( -R^{(9)} + h^{-1} \pi^{MN} \pi_{MN} - \frac{1}{8} h^{-1} \pi^2 \right) - 2h^{1/2} \eta_N \nabla_M (h^{-1/2} \pi^{MN}) \quad (2.4)$$

Notice that the compensating vectors  $\eta^N = \eta_I^N \dot{u}^I$  only appear as Lagrange multipliers, enforcing the constraints

$$D_N \left( h^{-1/2} \pi^{NM} \right) = 0. \quad (2.5)$$

After satisfying this, one can choose the gauge  $\eta^N = 0$ , as usual in constrained Hamiltonian systems. Therefore, in the Hamiltonian framework, their dynamical role is manifest.

Notice that the time variation  $\dot{h}_{MN}$  always appears combined with the Lagrange multipliers, as in (2.3). For this reason, it is convenient to introduce the new metric fluctuation

$$\delta_I h^{MN} := \frac{\partial h^{MN}}{\partial u^I} - D^M \eta_I^N - D^N \eta_I^M. \quad (2.6)$$

Similarly, from the canonical momentum we define the variation

$$\dot{u}^I \delta_I \pi^{MN} := 2(g_{tt})^{1/2} \left( h^{-1/2} \pi^{MN} \right) = \dot{u}^I \left( \delta_I h^{MN} - h^{MN} \delta_I h \right). \quad (2.7)$$

The Hamiltonian constraint now becomes

$$D_N \left[ (g^{tt})^{1/2} \delta_I \pi^{MN} \right] = 0, \quad (2.8)$$

which implies that  $\delta_I \pi^{MN}$  is orthogonal to gauge transformations. The kinetic term extracted from the Hamiltonian reads

$$H_{\text{kin}} = \frac{1}{4} \dot{u}^I \dot{u}^J \left( \int d^{D-1}x \sqrt{-g_D} g^{tt} \left[ \delta_I \pi_{MN} \delta_J \pi^{MN} - \frac{1}{D-2} \delta_I \pi \delta_J \pi \right] \right), \quad (2.9)$$

with  $D = 10$  in the present case. The kinetic term is entirely determined by  $\delta_I \pi^{MN}$ , which is then interpreted as the 10d gauge invariant metric fluctuation corresponding to the zero mode  $u^I(x)$ .

## 2.2 Kinetic terms

Performing the dimensional reduction starting from (2.9), the constraint along the 4d directions sets  $\delta_I \pi^{\mu\nu} = 0$ , which is equivalent to

$$\delta_I e^{2A} = -\frac{1}{2} e^{2A} g^{kl} \delta_I g_{kl}. \quad (2.10)$$

Then the warp factor variation may be eliminated from  $\delta \pi_{ij}$  yielding,

$$\delta_I \pi_{ij} = \delta_I g_{ij} + \frac{1}{2} g_{ij} g^{kl} \delta_I g_{kl}. \quad (2.11)$$

The constraint along the internal directions,

$$D^N ((g^{tt})^{1/2} \delta_I \pi_{Nj}) = 0, \quad (2.12)$$

implies that the physical fluctuation  $\delta_I \pi_{ij}$  is in harmonic gauge with respect to the full 10d warped metric.

With these results, the field space metric becomes [1]

$$G_{IJ}(u) = \frac{1}{4} \int d^6y \sqrt{g_6} e^{2A} \left( \delta_I \pi_{ij} \delta_J \pi^{ij} - \frac{1}{8} g^{ij} \delta_I \pi_{ij} g^{kl} \delta_J \pi_{kl} \right). \quad (2.13)$$

The metric  $G_{IJ}$  is given as an inner product (depending explicitly on the warp factor) between tangent vectors  $\delta_I \pi_{ij}$  and  $\delta_J \pi_{ij}$ . The condition (2.12) implies that the physical variation is orthogonal to gauge transformations. An equivalent statement is that the constraint equation minimizes the inner product over each gauge orbit. This is exactly what happens in the simpler Yang-Mills case, where the canonical momentum is the electric field, the constraint is Gauss's law, and the kinetic term is proportional to the electric energy.

This method applies to general warped compactifications preserving 4d maximal symmetry. No assumptions about the internal metric  $g_{ij}(y)$  in eq. (2.1) are required. The application of these results to the particular case of a conformal Calabi-Yau metric (1.5) is discussed in [1] and will be summarized in section 3.3.



### 3. Finding the universal volume modulus

Our aim is to find the 10d solution describing a finite spacetime dependent fluctuation of the volume modulus. Now, as explained in section 1.1, the first problem one faces is that of defining the volume modulus in warped backgrounds. We address this issue by finding the modulus in the case of an infinitesimal fluctuation, and then showing how to integrate it to a finite variation in section 6.

Before proceeding, we should clarify the type of expansion being performed. One starts from a warped background of the general form (2.1), where  $\hat{g}_{\mu\nu}$  is a maximally symmetric 4d metric. Then, a given modulus  $u$  is allowed to have a nontrivial spacetime dependence, acquiring a nonzero velocity  $\dot{u}$  and energy  $g^{tt}(\dot{u})^2$ . The energy sources the Ricci tensor, with the result that maximal symmetry is lost; for instance, for a massless excitation we would have a pp-wave spacetime. The important point is that backreaction is proportional to the energy, and hence is quadratic in  $\dot{u}$ . The linearized expansion we consider here then means working at first order in moduli velocities, so that the 4d metric can still be approximated by a maximally symmetric space. In this limit, the metric fluctuations  $\dot{h}_{MN} = \dot{u}^I \partial_I h_{MN}$  amount to a small perturbation around the background solution  $h_{MN}$  even if  $\partial_I h_{MN}$  is not necessarily small. This is enough for the purposes of finding the Kähler potential.

We apply the Hamiltonian approach to find the linearized 10d wavefunctions of the universal volume modulus (in this section) and its axionic partner (in section 4). These results will be used in section 5 to compute the Kähler potential. Finally, in section 6 we extend our results beyond the linear approximation, finding the backreaction produced by a finite volume modulus fluctuation. We restrict to type IIB with BPS fluxes and branes [4–7], so that the internal manifold is conformally equivalent to a Calabi-Yau:

$$ds^2 = e^{2A_0(y)} \hat{\eta}_{\mu\nu} dx^\mu dx^\nu + e^{-2A_0(y)} \tilde{g}_{ij}(y) dy^i dy^j \quad (3.1)$$

(thus, we work in the orientifold limit with constant axio-dilaton as well). It would be interesting to apply our approach to general  $\mathcal{N} = 1$  flux compactifications.

#### 3.1 Ten dimensional wavefunction

Consider an ansatz of the form (2.2),

$$ds^2 = e^{2A(y;c)+2\Omega[c]} (\hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + 2 \partial_j B \partial_\mu c dx^\mu dy^j) + e^{-2A(y;c)} \tilde{g}_{ij}(y) dy^i dy^j, \quad (3.2)$$

where  $c(x)$  denotes the universal volume modulus. As will be seen momentarily, a compensating field proportional to a total derivative,

$$\eta_j(y) = e^{2A+2\Omega} \partial_j B(y), \quad (3.3)$$

solves the Hamiltonian constraints, so we have already made this identification in the ansatz. The Weyl factor is defined to bring us to 4-dimensional Einstein frame,

$$e^{2\Omega(c)} = \frac{\int d^6 y \sqrt{\hat{g}_6}}{\int d^6 y \sqrt{\tilde{g}_6} e^{-4A(y;c)}} = \frac{V_{CY}}{V_W(c)}. \quad (3.4)$$

Furthermore, the underlying CY metric is taken to be independent of the volume modulus because a rescaling  $\tilde{g}_{ij} \rightarrow \lambda \tilde{g}_{ij}$  amounts to a 4d Weyl transformation.

At the end of the section it will be argued that  $c(x)$  is actually orthogonal to the other non-universal metric zero modes  $u^I(x)$ . It is then consistent to set these to zero in the present discussion. Next we will show how the Hamiltonian approach determines the 10d wavefunction (3.2). The full computation is somewhat technical, so in section 3.2 we summarize the results.

The first step is to compute the canonical momentum (2.7) associated to the ansatz eq. (3.2). These are found to be

$$\begin{aligned} \delta_c \pi_{\mu\nu} &= 2 h_{\mu\nu} \left( 4 \frac{\partial A}{\partial c} - 2 \frac{\partial \Omega}{\partial c} + \nabla^i \eta_i + 2 \partial^i A \eta_i \right) \text{ and} \\ \delta_c \pi_{ij} &= g_{ij} \left( 4 \frac{\partial A}{\partial c} - 6 \frac{\partial \Omega}{\partial c} + 2 \nabla^i \eta_i + 6 \partial^i A \eta_i \right) - \nabla_i \eta_j - \nabla_j \eta_i, \end{aligned} \quad (3.5)$$

where  $\eta_i$  is given in (3.3).

Since  $\delta_c \pi_{\mu\nu}$  is proportional to  $h_{\mu\nu}$ , the constraint  $D^N \pi_{N\nu} = 0$  sets  $\partial^\mu (\delta_c \pi_{\mu\nu}) = 0$ . This relation should be valid for arbitrary  $\partial_\mu c$ , implying  $\delta_c \pi_{\mu\nu} = 0$ , or

$$4 \frac{\partial A}{\partial c} - 2 \frac{\partial \Omega}{\partial c} + e^{2\Omega+4A} \tilde{\nabla}^2 B = 0, \quad (3.6)$$

in terms of the derivative  $\tilde{\nabla}_i$  and Laplacian compatible with  $\tilde{g}_{ij}$ .

The constraint  $D^N \pi_{Nj} = 0$  requires a bit more of work. Fortunately, we can use the computation of the Ricci tensor component  $R_{\mu i}$  in [23, 14] for our purposes, recalling the relation [32]

$$R_{0i} = -D^N (h^{-1/2} \pi_{Ni}). \quad (3.7)$$

(We also need a diffeomorphism transformation to set  $\eta_i = 0$  and  $\eta_\mu = -e^{2A+2\Omega} \partial_\mu \dot{c} B$ , which can always be done for a compensator of the form (3.3)). The constraint then sets

$$\partial_m (\partial_c e^{-4A(y;c)}) = 0, \quad (3.8)$$

which implies that the dependence of the warp factor on  $c(x)$  is given by an additive shift

$$e^{-4A(y;c)} = e^{-4A_0(y)} + c(x), \quad (3.9)$$

where  $e^{-4A_0(y)}$  denotes the solution associated to the metric  $\tilde{g}_{ij}$ , which is independent of  $c(x)$ . A possible multiplicative factor is fixed using the integrated version of (3.6).

This result has an intuitive interpretation. In conformally CY flux compactifications, the background equations of motion only fix  $e^{-4A}$  up to a shift  $e^{-4A} \rightarrow e^{-4A} + c$ . It was noticed in [23, 24, 14] that a change in  $c$ , which is not a simple metric rescaling, also changes the internal volume, leading to the proposal that  $c$  represents the time-independent universal volume modulus. What we find here is that this shift is present in the full time-dependent case too, although the full 10d metric fluctuation has other components as well.

Finally, plugging (3.4) and (3.9) into (3.6), we obtain the differential equation that fixes the compensating field (also observed in [14]),

$$\tilde{\nabla}^2 B = -e^{-4A-2\Omega} \left( 4 \frac{\partial A}{\partial c} - 2 \frac{\partial \Omega}{\partial c} \right) = -e^{-4A_0} + \frac{V_W^0}{V_{\text{CY}}}, \quad (3.10)$$

where  $V_W^0 = \int d^6 y \sqrt{\tilde{g}_6} e^{-4A_0(y)}$  is the background value of the warped volume. This equation is consistent in compact CY manifolds because the right hand side integrates to zero (which is actually the condition which fixes the factor of  $e^{2\Omega}$  in (3.3)). Therefore, the 10d metric solving the Hamiltonian constraints,

$$ds_{10}^2 = \left[ e^{-4A_0(y)} + c(x) \right]^{-1/2} e^{2\Omega[c(x)]} \left( \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + 2 \partial_i B \partial_\mu c dy^i dx^\mu \right) + \left[ e^{-4A_0(y)} + c(x) \right]^{1/2} \tilde{g}_{ij}(y) dy^i dy^j \quad (3.11)$$

gives a consistent spacetime dependent solution representing infinitesimal fluctuations of the universal volume modulus. The last part of the 10d fluctuation is in the 4-form potential, which is proportional to  $e^{4A}$ . Intuitively, the BPS-like condition of [7] sets  $C_4 = e^{4\Omega} e^{4A(y;c)} d^4 x$ , so the 4-form fluctuates along with the volume modulus. More details are given in section 6, where these results will be extended to finite fluctuations.

### 3.2 Summary

Briefly summarizing the main points of the previous computation, the warped universal volume modulus is not associated to a simple trace rescaling of the underlying CY metric, unlike in the unwarped case. Rather,  $\tilde{g}_{ij}(y)$  stays fixed and the modulus corresponds to an additive shift

$$e^{-4A(x,y)} = e^{-4A_0(y)} + c(x), \quad (3.12)$$

where  $e^{-4A_0(y)}$  is the background solution with respect to  $\tilde{g}_{ij}$ . There is also a nonzero compensating field  $\partial_i B$  determined by (3.10).

A more physical way of stating this is by noticing that in the 4d action the compensating field *only* appears through the shift [1]

$$\delta_c g_{MN} = \frac{\partial g_{MN}}{\partial c} - D_N (e^{2A+2\Omega} \partial_M B) - D_M (e^{2A+2\Omega} \partial_N B), \quad (3.13)$$

where  $D_N$  is the covariant derivative with respect to the 9d spacelike metric (see section 2). The physical 10d fluctuation associated to  $c(x)$  then becomes

$$\delta_c g_{\mu\nu} = 2 e^{2A+2\Omega} \eta_{\mu\nu} \left( \delta_c A + \frac{\partial \Omega}{\partial c} \right), \quad \delta_c g_{ij} = -e^{-2A} (2 \delta_c A \tilde{g}_{ij} + \delta_c \tilde{g}_{ij}), \quad (3.14)$$

where

$$\delta_c A := \frac{\partial A}{\partial c} - e^{4A+2\Omega} \partial^i A \partial_i B, \quad \delta_c \tilde{g}_{ij} = \tilde{\nabla}_i [e^{4A+2\Omega} \partial_j B] + \tilde{\nabla}_j [e^{4A+2\Omega} \partial_i B]. \quad (3.15)$$

The dependence of  $\Omega$  and  $A$  on  $c(x)$  is given in (3.4) and (3.9). Strikingly, for non-trivial warping the universal volume modulus has an internal metric fluctuation  $\delta_c g_{ij}$  which *is*

not pure trace. The nontrivial dependence comes from the effect of the compensating field. Stated in gauge invariant terms, this is required so that the canonical momentum  $\delta_c \pi_{MN}$  built from  $\delta_c g_{MN}$  is in harmonic gauge with respect to the warped 10d metric.

Notice that in the unwarped (or large volume) limit the warp factor becomes  $e^{-4A} \approx c(x) := e^{4u(x)}$ , which in turn implies  $e^{2\Omega} = e^{-4u(x)}$ . The equation of motion for the compensator (3.10) becomes simply  $\tilde{\nabla}^2 B = 0$ , which is solved by  $B = 0$ , so we regain the usual metric for the universal volume modulus in the unwarped case (1.2).

### 3.3 Orthogonality with other modes

The metric moduli arise as independent solutions to a Sturm-Liouville problem. Different zero modes should be orthogonal to each other, and we may use this to understand how to define the universal volume modulus from the original  $h^{1,1}$  moduli.

The natural inner product is given by the Hamiltonian (2.9). Consider two zero mode solutions, with canonical momenta  $\delta_I \pi_{MN}$  and  $\delta_J \pi_{MN}$  respectively ( $I \neq J$ ). The orthogonality condition reads

$$G_{IJ} = \int d^{D-1} x \sqrt{-g_D} g^{tt} [\delta_I \pi_{MN} \delta_J \pi^{MN} - \frac{1}{D-2} \delta_I \pi \delta_J \pi] = 0, \quad (3.16)$$

where  $D = 10$  in our case.

We need to compute the inner product (3.16) between the universal volume modulus and the nonuniversal metric fluctuations. Recall that the canonical momentum associated to such a fluctuation is [1]

$$\delta_I \pi_{ij} = e^{-2A} \left( \delta_I \tilde{g}_{ij} - \frac{1}{2} \tilde{g}_{ij} \delta_I \tilde{g} \right), \quad (3.17)$$

where

$$\delta_I \tilde{g}_{ij} = \frac{\partial \tilde{g}_{ij}}{\partial u^I} - \tilde{\nabla}_i (e^{4A} B_{Ij}) - \tilde{\nabla}_j (e^{4A} B_{Ii}). \quad (3.18)$$

Here  $B_{Ij}$  is the compensating field required by the time-dependent fluctuation  $\partial \tilde{g}_{ij} / \partial u^I$ . Unlike the case of the universal modulus, the  $B_{Ij}$  are not total derivatives; compare with eq. (3.14) and eq. (3.15).

Next, specialize to  $I = c$ , the universal volume modulus, and  $J \neq c$  a nonuniversal zero mode. Using orthogonality with respect to gauge transformations and  $\delta \pi_{\mu\nu} = 0$ ,

$$G_{cJ} = \int d^{D-1} x \sqrt{-g_D} g^{tt} \frac{\partial g^{ij}}{\partial c} \delta_J \pi_{ij}. \quad (3.19)$$

Recalling that  $\partial g_{ij} / \partial c = (1/2) e^{4A} g_{ij}$ , the orthogonality condition requires

$$\int d^6 y \sqrt{\tilde{g}_6} \tilde{g}^{ij} \delta_J \tilde{g}_{ij} = 0, \quad (3.20)$$

which is solved by

$$\tilde{g}^{ij} \frac{\partial \tilde{g}_{ij}}{\partial u^J} = 0. \quad (3.21)$$

The compensating fields in (3.18) drop from (3.20), being total derivatives.

The nonuniversal Kähler moduli thus correspond to the  $h^{1,1} - 1$  *traceless* combinations, and eq. (3.21) defines the basis of linearly independent metric zero modes orthogonal to the universal volume modulus. It is interesting that we recover the known result from CY compactifications, although the universal mode is no longer a pure trace fluctuation of the internal metric. We should also point out that (3.21) is not a gauge condition: we can fix completely the diffeomorphism redundancies by setting the compensating fields to zero, but we would still need to impose (3.21). Rather, it tells us how to choose a particular basis in the space of solutions to the Sturm-Liouville problem of the metric zero modes. This grants that there are no kinetic mixings between the volume modulus and the other zero modes.

#### 4. Axionic partner of the volume modulus

In the unwarped limit, the universal volume modulus gets complexified with the axion coming from

$$C_4 = \frac{1}{2} a(x) \tilde{J}(y) \wedge \tilde{J}(y) + \dots \quad (4.1)$$

In this section, we construct the universal axion in warped backgrounds. This will be the partner of the warped volume modulus (3.14). At the end of the section, the  $h^{1,1} - 1$  nonuniversal axions will be shown to be orthogonal to the universal axion, so they will be set to zero in the main part of the analysis. This is the counterpart of what happens with the universal volume modulus, as can be anticipated for supersymmetric compactifications.

The Hamiltonian formulation for antisymmetric tensors is similar to the familiar U(1) Maxwell case, where the canonical momentum is the electric field,

$$E^i = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = g^{tt} g^{ij} (\partial_0 A_j - \partial_j A_0) \quad (4.2)$$

and  $A_0$  is a Lagrange multiplier enforcing Gauss's law  $\nabla^i E_i = 0$ . The shift of (4.2) by  $\partial_i A_0$  is the analog of the metric fluctuation shift (2.6) by the compensating field.

The generalization to a  $p$ -form  $C_p$  is as follows.  $C_{0i_2 \dots i_p}$  plays the role of a Lagrange multiplier, and the canonical momentum is given by the  $p + 1$ -form

$$E := \frac{1}{(p+1)!} F_{0i_1 \dots i_p} dx^0 \wedge dx^{i_1} \wedge \dots \wedge dx^{i_p} \quad (4.3)$$

where  $i_1, \dots, i_p$  are spacelike indices. If there are no couplings to external fields the constraint is

$$d(\star_D E) = 0 \quad (4.4)$$

The Hamiltonian kinetic term is then

$$\int dt H_{\text{kin}} = \int E \wedge \star_D E = \frac{1}{(p+1)!} \int d^D x \sqrt{g_D} F_{0i_1 \dots i_p} F^{0i_1 \dots i_p} \quad (4.5)$$

This is gauge invariant due to (4.4). The magnetic field contributions  $F_{i_1 \dots i_{p+1}}$  appear in the potential energy.

#### 4.1 Axion fluctuation in a warped background with flux

We now apply the previous approach to find the 10d universal axion in the warped background (3.1) with three-form flux; there are extra subtleties arising from self-duality and the unusual gauge transformations of the 4-form potential. We start by generalizing the known form from unwarped compactifications, since the wavefunction should reduce to that form in the unwarped limit. We also find that a constant axion yields a trivial field strength, even in the presence of a fluctuating volume modulus, so the solution respects the classical axion shift symmetry. Also, recall that we are working in the limit of constant axio-dilaton.

Because the 4-form potential transforms under gauge transformations associated with the 2-form potentials, there is a small subtlety in determining 4-form fluctuations that are globally defined on the compactification [9]. We discuss the details in appendix A; we will find that, in terms of globally defined fluctuations, the 5-form and 3-form canonical momenta (4.3) are

$$\tilde{E}_5 = d\delta C_4 + \frac{ig_s}{2}(\delta A_2 \wedge \bar{G}_3 - \delta \bar{A}_2 \wedge G_3) \quad (4.6)$$

$$E_3 = d\delta A_2, \quad (4.7)$$

where  $A_2 = C_2 - \tau B_2$ .  $\delta C_4$  and  $\delta A_2$  denote the components of  $C_4$  and  $A_2$  which depend on the axion field; their explicit form will be given momentarily. The presence of the “transgression terms” in (4.6) reflects the fact that the canonical momenta are invariant under the gauge transformations

$$\begin{aligned} \delta C_4 &\rightarrow \delta C_4 + d\chi_3 + \frac{ig_s}{2}(\bar{\zeta}_1 \wedge G_3 - \zeta_1 \wedge \bar{G}_3), \\ \delta A_2 &\rightarrow \delta A_2 + d\zeta_1. \end{aligned} \quad (4.8)$$

We expect the axion to descend from the 4-form gauge potential  $\delta C_4$ ; however, we notice that there are two separate gauge transformations associated with  $\delta C_4$ , one of which arises from gauge transformations of  $\delta A_2$ . From the Hamiltonian perspective, gauge transformations are associated with corresponding compensators, so we expect that there should be compensators for the axion associated with *both*  $\delta C_4$  and  $\delta A_2$ .

We take the ansatz

$$\delta C_4 = \frac{1}{2}a_0(x)\tilde{J}^2 + a_2(x) \wedge \tilde{J} - da_0 \wedge K_3 - da_2 \wedge K_1, \quad \delta A_2 = -da_0 \wedge \Lambda_1 \quad (4.9)$$

(note that  $\tilde{J} \wedge \tilde{J} = 2\tilde{\kappa}_6\tilde{J}$ ). Here,  $a_0$  and  $a_2$  are spacetime 0- and 2-forms respectively, while  $K_{1,3}$  and  $\Lambda_1$  are forms on the internal manifold included as possible compensators. The canonical momenta (4.6-4.7) are then

$$\tilde{E}_5 = da_0 \wedge \left( \tilde{\kappa}_6\tilde{J} + dK_3 - \frac{ig_s}{2}\Lambda_1 \wedge \bar{G}_3 + \frac{ig_s}{2}\bar{\Lambda}_1 \wedge G_3 \right) + da_2 \wedge \left( \tilde{J} + dK_1 \right) \quad (4.10)$$

$$E_3 = da_0 \wedge d\Lambda_1. \quad (4.11)$$

Notice that  $\tilde{E}_5$  vanishes trivially for a constant axion  $a_0$ , so the field space metric cannot depend on the axion, as expected from the classical axion shift symmetry. The 5-form

canonical momentum  $\tilde{E}_5$  is self-dual, which reduces the 4d degrees of freedom to a single scalar by requiring  $da_2 \propto \hat{\star}_4 da_0$ . At linear order, the proportionality constant may depend only on expectation values of moduli (at higher orders, it may also depend on fluctuations of moduli); we will see that the full wavefunction requires the choice  $da_2 = e^{4\Omega} \hat{\star}_4 da_0$ . In this work we only keep  $a_0$  as an independent field, multiplying the kinetic term by 2.<sup>2</sup>

Imposing the constraint (4.4) for the 5-form, we find that

$$d \left[ e^{4A} \left( \tilde{J} + \tilde{\star}_6 \left( dK_3 - \frac{ig_s}{2} \Lambda_1 \wedge \bar{G}_3 + \frac{ig_s}{2} \bar{\Lambda}_1 \wedge G_3 \right) \right) \right] = 0 \quad (4.12)$$

$$d \left[ e^{-4A} \left( \tilde{\star}_6 \tilde{J} + \tilde{\star}_6 dK_1 \right) \right] = -\frac{ig_s}{2} e^{-2\Omega} (d\Lambda_1 \wedge \bar{G}_3 - d\bar{\Lambda}_1 \wedge G_3) . \quad (4.13)$$

These constraints are identical to the 10d equations of motion  $d(\star_{10} \tilde{F}_5) = (ig_s/2) G_3 \wedge \bar{G}_3$  evaluated for legs in the internal directions. (The factor of  $e^{-2\Omega}$  on the right-hand-side of eq. (4.13) is related to the proportionality factor in the 4d Poincaré duality between  $a_0$  and  $a_2$ .) In this way, the Hamiltonian and Lagrangian approaches yield equivalent results, and  $a_0$  corresponds to a massless 4d field.

For the volume modulus, the compensating field is determined by a single scalar function  $\eta_i = e^{2A+2\Omega} \partial_i B$ , and we expect the same to occur for the compensator in  $\delta C_4$ . The form of the compensator equation (4.12) then motivates the following ansatz,

$$e^{4A} \left[ \tilde{J} + \tilde{\star}_6 \left( dK_3 - \frac{ig_s}{2} \Lambda_1 \wedge \bar{G}_3 + \frac{ig_s}{2} \bar{\Lambda}_1 \wedge G_3 \right) \right] = e^{2\Omega} \tilde{J} + e^{2\Omega} d(e^{4A} dK) \quad (4.14)$$

in terms of a function  $K(y)$ . The factor of  $e^{2\Omega}$  is fixed by wedging (4.14) with  $\tilde{\star}_6 \tilde{J}$  and integrating over the internal space. In fact, this ansatz yields an appropriately self-dual 5-form if we take  $K_1 = e^{4A} dK$ , and the factor here precisely fixes the proportionality in the relation between  $a_0$  and  $a_2$ . Replacing this ansatz in (4.13), we obtain the compensator equation for  $K(y)$ ,

$$d(\tilde{\star}_6 [dA \wedge dK]) + \frac{1}{8} de^{-4A} \wedge \tilde{J} \wedge \tilde{J} = -e^{-2\Omega} \frac{ig_s}{8} (d\Lambda_1 \wedge \bar{G}_3 - d\bar{\Lambda}_1 \wedge G_3) . \quad (4.15)$$

The second constraint, associated with the  $A_2$  gauge transformation, fixes the compensator  $\Lambda_1$ ,

$$d(\tilde{\star}_6 d\Lambda_1) = -4i e^{2\Omega} e^{4A} dA \wedge dK \wedge G_3 . \quad (4.16)$$

This follows from the  $G_3$  equation of motion, primitivity of the 3-form,<sup>3</sup> and the 4d Poincaré duality relation (which fixes the power of  $e^{2\Omega}$ ).

There is one other issue in this analysis. Because there is a background 5-form associated with the warp factor, the axion fluctuations can appear in the Hamiltonian equation for  $\dot{\pi}_{MN}$  at linear order, through terms of the form  $\delta \tilde{F}_{MP_1 \dots P_4} \tilde{F}_N^{P_1 \dots P_4}$ . By examining the allowed components, we can see that the only terms that contribute are of the form

$$4\tilde{F}_\mu^{\nu\lambda\rho n} \delta \tilde{F}_{m\nu\lambda\rho n} + \tilde{F}_m^{npqr} \delta \tilde{F}_{\mu npqr} . \quad (4.17)$$

<sup>2</sup>See [33] for a careful treatment of the self-dual form.

<sup>3</sup>On an orientifold  $T^6$  or  $T^2 \times K3$ , an additional term may appear in eq. (4.16) if the flux breaks supersymmetry, but it cancels out of the following analysis.

However, with the background 4-form potential proportional to the 4d volume form, self-duality of the 5-form causes this contribution to vanish for any fluctuations  $\delta\tilde{F}$  with these components.

Summarizing, the gauge invariant wavefunction for the universal axion in a warped background is given by the canonical momenta

$$\tilde{E}_5 = (1 + \star_{10}) \left[ e^{2\Omega} da_0(x) \wedge \tilde{\star}_6 \left( e^{-4A} \tilde{J} + 4 dA \wedge dK \right) \right] \quad (4.18)$$

$$E_3 = da_0 \wedge d\Lambda_1, \quad (4.19)$$

where  $K, \Lambda_1$  satisfy the Gauss law constraints (4.15), (4.16) respectively. Heuristically, the warp factor dependence arises naturally from  $J \wedge J = e^{-4A} \tilde{J} \wedge \tilde{J}$ .

In the unwarped limit, we see that the compensators become gauge trivial. First,  $K_1$  becomes exact. Similarly eq. (4.16) implies that  $\Lambda_1$  is closed, so  $\delta G_3 = 0$ . The residual gauge freedom to make  $\Lambda_1$  co-closed means that it must vanish (because there are no harmonic 1-forms on a CY); this same gauge transformation also forces  $K_3$  to be closed, as required by eq. (4.14) since  $e^{2\Omega} = e^{4A} = c^{-1}$ . Then it is simple to gauge away the  $K_1$  and  $K_3$  compensators in eq. (4.9). As expected, we then recover the known axion wavefunction in a CY background. Also note that the compensators  $\Lambda_1$  become trivial when the background 3-form flux vanishes, which we expect because  $\delta C_4$  has only one gauge transformation in that case.

## 4.2 Orthogonality with nonuniversal axions

We now consider the effect of the  $h^{1,1} - 1$  nonuniversal axions. The story is similar to the above. For  $\tilde{\rho}_r$  the independent  $(1, 1)$  forms in the 2nd cohomology ( $\tilde{J} = \tilde{\rho}_1$ ), the potential now becomes

$$\delta C_4 = \sum_{r=1}^{h^{1,1}} [a_0^r(x) \tilde{\star}_6 \tilde{\rho}_r + a_2^r(x) \wedge \tilde{\rho}_r - da_0^r \wedge dK_{3,r} - da_2^r \wedge dK_{1,r}], \quad \delta A_2 = -da_0^r \wedge \Lambda_{1,r}. \quad (4.20)$$

Computing the canonical momentum, we obtain constraints analogous to (4.12), (4.13), which along with self-duality imply

$$e^{4A} \left( \tilde{\rho}_r + \tilde{\star}_6 dK_{3,r} + \frac{ig_s}{2} \Lambda_{1,r} \wedge \bar{G}_3 - \frac{ig_s}{2} \bar{\Lambda}_{1,r} \wedge G_3 \right) = e^{2\Omega} M_r^s(u) (\tilde{\rho}_s + dK_{1,s}), \quad (4.21)$$

with  $M(u)$  some function of the moduli  $u^I$ , which can be diagonalized. The constraint from the 2-form gauge transformation is of a similar form as (4.16), but with a more general 1-form  $K_1 \neq e^{4A} dK$  on the right hand side, because there are no harmonic 5-forms on a CY.<sup>4</sup>

The kinetic term mixing between the universal and nonuniversal axions is

$$\int \tilde{E}_{5,r} \wedge \star_{10} \tilde{E}_{5,1} + \frac{g_s}{2} \int E_{3,r} \wedge \star_{10} \bar{E}_{3,1} + \frac{g_s}{2} \int \bar{E}_{3,r} \wedge \star_{10} E_{3,1} \quad (4.22)$$

---

<sup>4</sup>Again, there are additional terms on  $T^6$  or  $T^2 \times K3$ , but they still cancel in the kinetic term.



Using the constraint equations (4.21), (4.16) in a calculation similar to that presented below in section 5, the kinetic mixing is proportional to  $(\tilde{\rho}_r, \tilde{J}) = \int \tilde{\kappa} \rho_r \wedge \tilde{J}$ . Since this is the natural inner product on the 2nd cohomology, the universal axion is orthogonal to the other  $h^{1,1} - 1$  axionic excitations as long as the basis of (1, 1) forms is chosen to be orthogonal itself.

## 5. Kähler potential

Finally we are ready to compute the kinetic term and Kähler potential for the chiral superfield

$$\rho = a_0 + i c \tag{5.1}$$

which combines the universal Kähler modulus found in eq. (3.14) with the axionic mode given in eq. (4.18). Finding an explicit answer for the Kähler potential is in general rather involved, because the compensating fields appear explicitly in the kinetic terms. Therefore, one would have to solve the second order constraint equations (which depend on the warp factor) and then plug in the explicit solution into the kinetic terms. However, using the Hamiltonian expressions for the kinetic terms, we will find that the explicit solution to the compensating fields is actually not needed. We show that the constraint equations are enough to eliminate the compensating fields from the 4d action. In this way, we compute the explicit Kähler potential.

### 5.1 Kinetic terms

First we look at the kinetic term for  $c(x)$ ,

$$S_{\text{kin}, c} = \frac{1}{\kappa_4^2} \int d^4x \sqrt{\hat{g}} G_{cc} \hat{g}^{\mu\nu} \partial_\mu c \partial_\nu c. \tag{5.2}$$

According to section 2,  $G_{cc}$  follows from replacing the canonical momentum conjugate to (3.14) in the Hamiltonian expression (2.13). A short computation reveals that

$$G_{cc} = \frac{1}{2V_{\text{CY}}} \int d^6y \sqrt{\hat{g}} e^{4\Omega+2A} [e^{-2A-2\Omega}(\partial_c \Omega + \partial_c A) - e^{2A}(\partial^{\tilde{m}} A)(\partial_m B)]. \tag{5.3}$$

Integrating by parts to get  $\tilde{\nabla}^2 B$  and replacing it by its constraint (3.6), the terms containing  $\partial_c A$  cancel, and  $\partial_c \Omega$  controls the kinetic term. The result is

$$G_{cc} = \frac{3}{4} e^{4\Omega} = \frac{3}{4} \left( \frac{V_{\text{CY}}}{c(x)V_{\text{CY}} + V_W^0} \right)^2, \tag{5.4}$$

showing the well-known factor of 3 for the kinetic term of the universal volume modulus. It is interesting that this factor arises from nontrivial cancellations of different warping corrections, which would not occur had we neglected the compensating field contribution.

To calculate the kinetic term for the universal axion, we take the prescription for the 5-form in which we double the coefficient of the  $\tilde{F}_5^2$  term in the action but consider only half the components. We will keep the terms including the scalar  $a_0$  as opposed to  $a_2$  (with

$a_0 = a_0(t)$ , this corresponds to keeping components of  $\tilde{F}_5$  with time indices). Replacing the axion fluctuation (4.18) into the kinetic action, we find<sup>5</sup>

$$\begin{aligned}
 S_{\text{kin, a}} &= -\frac{1}{4\kappa_{10}^2} \int \left( \tilde{E}_5 \wedge \star_{10} \tilde{E}_5 + g_s E_3 \wedge \star_{10} \tilde{E}_3 \right) \\
 &= -\frac{1}{4\kappa_{10}^2} \int e^{4\Omega} da_0 \wedge \hat{\star}_4 da_0 \int \left[ \left( \tilde{\star}_6 (e^{-4A} \tilde{J} + 4dA \wedge dK) \right) \wedge \left( \tilde{J} + de^{4A} \wedge dK \right) \right. \\
 &\quad \left. + e^{-2\Omega} g_s d\Lambda_1 \wedge \tilde{\star}_6 d\bar{\Lambda}_1 \right]. \tag{5.5}
 \end{aligned}$$

Note that the Chern-Simons term does not include  $a_0$ , so it does not appear. Integrating by parts and using the constraint equations (4.15), (4.16) to eliminate the compensators  $K(y), \Lambda_1(y)$ , we arrive to

$$S_{\text{kin, a}} = -\frac{3}{4\kappa_4^2} \int \sqrt{-\hat{g}} e^{4\Omega} \hat{g}^{\mu\nu} \partial_\mu a_0 \partial_\nu a_0. \tag{5.6}$$

The factor of 3 comes from

$$\int \tilde{J} \wedge \tilde{\star}_6 \tilde{J} = \frac{1}{2} \int \tilde{J}^3 = 3V_{\text{CY}}. \tag{5.7}$$

This reproduces precisely the field space metric of the volume modulus. As we saw with the metric volume modulus, we see that the presence of the compensators in (5.5) are crucial to obtain the correct form for the kinetic term (5.6).

## 5.2 Kähler potential and no-scale structure

The previous analysis shows that the volume modulus and universal axion can be complexified into

$$\rho(x) = a_0(x) + i c(x). \tag{5.8}$$

In fact, since our analysis has not relied on the particular components of the 3-form flux, the volume modulus and axion form a complex scalar even in compactifications with classically broken supersymmetry. From the kinetic terms (5.2) and (5.6), we obtain

$$S_{\text{kin}} = -3 \frac{1}{\kappa_4^2} \int d^4x \sqrt{\hat{g}_4} \frac{\hat{g}^{\mu\nu} \partial_\mu \rho \partial_\nu \bar{\rho}}{\left[ -i(\rho - \bar{\rho}) + 2 V_W^0 / V_{\text{CY}} \right]^2} \tag{5.9}$$

This metric follows from the Kähler potential,

$$K = -3 \log \left( -i(\rho - \bar{\rho}) + 2 \frac{V_W^0}{V_{\text{CY}}} \right). \tag{5.10}$$

Corrections due to warping amount to an additive constant in the Kähler potential. This proves that no-scale structure  $G^{\rho\bar{\rho}} \partial_\rho K \partial_{\bar{\rho}} K = 3$  is maintained in GKP type compactifications, albeit in terms of a highly nontrivial 10d wavefunction for  $\rho$ . We can also write this Kähler potential in a more physical way in terms of the full warped volume,

$$K = -3 \log (2V_W(\rho)/V_{\text{CY}}). \tag{5.11}$$

---

<sup>5</sup>Recall that  $E_p$  is the “electric field”  $F_{0i_1 \dots i_{p-1}}$ .

The quantity  $(V_W^0/V_{CY})$  may be interpreted as the background value for  $c(x)$ , so, after shifting

$$\rho \rightarrow \rho - i \frac{V_W^0}{V_{CY}}, \tag{5.12}$$

the Kähler potential is

$$K = -3 \log [-i(\rho - \bar{\rho})]. \tag{5.13}$$

This result coincides with the unwarped expression. The correction from warping becomes important, for example, once a nonperturbative superpotential for  $\rho$  is included as in [25]. The instanton or gaugino condensation superpotential receives then an exponential correction from warping due to the tree-level shift,<sup>6</sup>

$$W = A e^{ia\rho} \rightarrow A e^{aV_W^0/V_{CY}} e^{ia\rho}. \tag{5.14}$$

Similarly, if we consider  $\alpha'$  corrections [26], the shift modifies the potential for the volume modulus. The modifications in both these cases deserve further study.

The fact that a series of rather subtle corrections conspire to give the very simple final result (5.10) suggests that there could be some underlying physical reason for this.<sup>7</sup> One way to understand this is to notice that (in the absence of contributions beyond classical supergravity) the 10d solution we have found preserves the shift symmetry  $e^{-4A} \rightarrow e^{-4A} + c(x)$ . This implies no-scale structure, which in turn restricts the Kähler potential to be of the general form

$$K(\rho, \bar{\rho}) = -3 \log [-i(\rho - \bar{\rho}) + a] + b. \tag{5.15}$$

Therefore, the shift-symmetry of the full solution protects the Kähler potential from significant warping corrections.

## 6. Nonlinear solution for fluctuating volume modulus

In this section, we present a complete, nonlinear solution to the 10d supergravity field equations corresponding to a wave of the universal volume modulus. Our solutions are appropriate for compactifications of the form discussed in [7]. For ease of presentation, we will work with the covariant equations of motion.

The external spacetime metric in the time-dependent background takes a pp-wave form, as is appropriate for a propagating massless field. As a brief review, the pp-wave metric has the form

$$ds_4^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = -H(x^+, \vec{x})(dx^+)^2 + dx^+ dx^- + d\vec{x}^2. \tag{6.1}$$

A clear but important property of this metric is that  $\hat{g}^{++} = 0$ . It will be important later that most of the Christoffel symbols vanish; in particular,  $\hat{\Gamma}_{\mu\nu}^+ = 0$ . The only nonvanishing Ricci tensor component is  $\hat{R}_{++} = (1/2)\vec{\partial}^2 H$ , so the Ricci scalar vanishes.

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<sup>6</sup>Here we are ignoring possible corrections to the Kähler potential in the  $\alpha'$  and  $g_s$  expansions, as well as nonperturbative corrections.

<sup>7</sup>We thank S. Kachru and A. Tomasiello for discussions on this point.

As a source, consider a massless scalar with action

$$S = -\frac{1}{2\kappa_D^2} \int d^D x \sqrt{-\hat{g}} f(\phi) (\partial\phi)^2 . \quad (6.2)$$

It is clear that any function  $\phi(x^+)$  solves the scalar equation of motion, and, since  $\partial\phi$  is null, the Einstein equation is (the only nontrivial component is  $++$ )

$$\hat{R}_{\mu\nu} = f(\phi) \partial_\mu \phi \partial_\nu \phi , \quad (6.3)$$

which is solved by

$$H(x^+, \vec{x}) = \frac{1}{2(D-2)} |\vec{x}|^2 f(\phi(x^+)) (\partial_+ \phi(x^+))^2 . \quad (6.4)$$

Since  $H$  is quadratic in the scalar velocity, we see immediately why previous attempts to solve for the volume modulus beyond linear order have failed.

### 6.1 Ten-dimensional solution

We can now present the nonlinear solution for a propagating volume modulus and verify that it solves the equations of motion. The warp factor profile in the compact dimensions remains the same as in the static case, and the compensator wavefunction is given by the linearized expression. In addition, since 3-form fluxes do not stabilize the volume modulus, we include the 3-forms quite simply, so these results apply to all GKP compactifications [7]. Throughout, we assume that 7-branes are in the orientifold limit, so that the internal space is conformally CY and the axio-dilaton is constant. We also work away from localized sources such as branes or orientifolds for simplicity; removing these assumptions is a straightforward generalization.

The 10d background corresponding to a finite fluctuation of the universal volume modulus can be written as

$$ds^2 = e^{2A(x,y)} e^{2\Omega(x)} \bar{g}_{\mu\nu}(x,y) dx^\mu dx^\nu + e^{-2A(x,y)} \tilde{g}_{ij}(y) dy^i dy^j \quad (6.5)$$

$$\tilde{F}_5 = e^{4\Omega} d^4 x \wedge d(e^{4A}) + \tilde{\star} d(e^{-4A_0}) , \quad (6.6)$$

where we have defined the shorthand  $e^{2\Omega}$  for the Einstein frame factor as in eq. (3.4) and the warp factor as in eq. (3.9) as well as a 4d metric

$$\bar{g}_{\mu\nu}(x,y) = \hat{g}_{\mu\nu}(x) - 2 \left( \hat{\nabla}_\mu \partial_\nu c(x) + e^{2\Omega(x)} \partial_\mu c(x) \partial_\nu c(x) \right) B(y) . \quad (6.7)$$

Here,  $\hat{g}_{\mu\nu}$  is a pp wave as defined in eq. (6.1), and  $B(y)$  is a compensator that obeys the same constraint as in the linear case eq. (3.10). In addition, the volume modulus  $c(x)$  depends only on a null direction, which we denote  $x^+$ . This means that  $\hat{\nabla}_\mu \partial_\nu c = \bar{\nabla}_\mu \partial_\nu c = \partial_+^2 c$  (or for any field). In addition, since  $\hat{g}_{\mu\nu}$  and  $\bar{g}_{\mu\nu}$  differ from Minkowski only in the  $++$  component,  $d^4 x$  is the volume form for those metrics as well (conveniently written in light-cone coordinates).

The first equation of motion to check is the 5-form Bianchi identity, which is satisfied as long as  $A_0$  is the appropriate static warp factor; with fixed background 3-form flux (and

local sources), the Bianchi identity is spacetime independent. Self-duality of the 5-form then fixes the spacetime component — the external component of  $C_4$  is just the volume form of the 4d spacetime. It is also easy to see that the axio-dilaton and 3-form equations of motion are unchanged from the static solution (up to overall factors), so they are trivially satisfied, as well.

We now proceed to the Einstein equation. The  $\mu i$  component is just the integrated form of eq. (3.8), which is satisfied by the “shifted” form (3.9) assumed. The internal component is slightly more complicated because it includes sources from the 5-form and 3-forms. However, because all 4d derivatives are null and the pp wave Ricci scalar vanishes, the Einstein equation reduces to the static case, which is satisfied by assumption. This is the Poisson equation

$$\tilde{\nabla}^2 e^{-4A_0} = -\frac{g_s}{12} G_{ijk} \tilde{G}^{ijk}, \quad (6.8)$$

which also follows from the 5-form Bianchi [14].

Finally, we consider the external components of the Einstein equation. A straightforward but somewhat tedious calculation finds the Ricci tensor

$$R_{\mu\nu} = \hat{R}_{\mu\nu} - 2\hat{\nabla}_\mu \partial_\nu \Omega + 4\hat{\nabla}_\mu \partial_\nu A + 2\partial_\mu \Omega \partial_\nu \Omega - 8\partial_{(\mu} \Omega \partial_{\nu)} A - 16\partial_\mu A \partial_\nu A - e^{2\Omega} e^{4A} \bar{g}_{\mu\nu} \tilde{\nabla}^2 A + e^{2\Omega} e^{4A} \left( \hat{\nabla}_\mu \partial_\nu c + e^{2\Omega} \partial_\mu c \partial_\nu c \right) B. \quad (6.9)$$

As in calculating the other components, we have made repeated use of the fact that all spacetime derivatives lie in the  $x^+$  direction, so contractions of them automatically vanish. The trace-reversed stress tensor (we take  $R_{MN} = T_{MN}$ ) has external components

$$T_{\mu\nu} = -4e^{2\Omega} e^{4A} (\partial_i A \partial^i A) \bar{g}_{\mu\nu} - \frac{g_s}{48} e^{2\Omega} e^{4A} G_{ijk} \tilde{G}^{ijk} \bar{g}_{\mu\nu}. \quad (6.10)$$

Then the external Einstein equation simplifies with the help of eq. (6.8) along with the relations (3.4), (3.9):

$$\hat{R}_{\mu\nu} + \hat{\nabla}_\mu \partial_\nu c \left[ e^{2\Omega} - e^{4A} + e^{2\Omega} e^{4A} \tilde{\nabla}^2 B \right] + \partial_\mu c \partial_\nu c \left[ -\frac{1}{2} e^{4\Omega} - e^{2\Omega} e^{4A} + e^{4\Omega} e^{4A} \tilde{\nabla}^2 B \right] = 0. \quad (6.11)$$

Since we take the compensator  $B$  to obey the constraint (3.10), we end up with

$$\hat{R}_{\mu\nu} = \frac{3}{2} e^{4\Omega} \partial_\mu c \partial_\nu c. \quad (6.12)$$

Note that the compensator term quadratic in  $c$  is necessary to cancel all the internal space dependence in the external Einstein equation. This is just the Einstein equation (6.3) for the 4d pp wave, as we desired.

## 6.2 Comments on the nonlinear background

Let us now make a few comments about the nonlinear background.

First, compare this background to the linearized one presented earlier. The Hamiltonian approach naturally defines the compensators as metric components  $g_{\mu i} \propto \partial_i B$ . These can be gauged away at the cost of introducing a deformation of the internal metric.

However, in the nonlinear solution, it is useful to work with coordinates in which  $\tilde{g}_{ij}$  is unchanged by the fluctuation and the compensator appears in the spacetime metric. In addition, the compensator now acquires a term quadratic in the modulus velocity. Finally, since the solution singles out the lightcone coordinate  $x^+$ , we found it more convenient to work with the covariant equations of motion. Otherwise, the nonlinear background is quite similar to the linearized one, and we see that the warp factor and compensator profiles are actually identical.

The existence of this nonlinear background has several important consequences. For one, the solution provides an independent derivation of the kinetic term for the volume modulus. That is, the 10d solution satisfies the 4d Einstein equation for the pp-wave (6.3), which exactly encodes the kinetic term for the massless scalar. In fact, we see that we reproduce the field space metric (5.4), even including the famous factor of 3. This fact is a highly nontrivial consistency check of the low energy theory that we have developed.

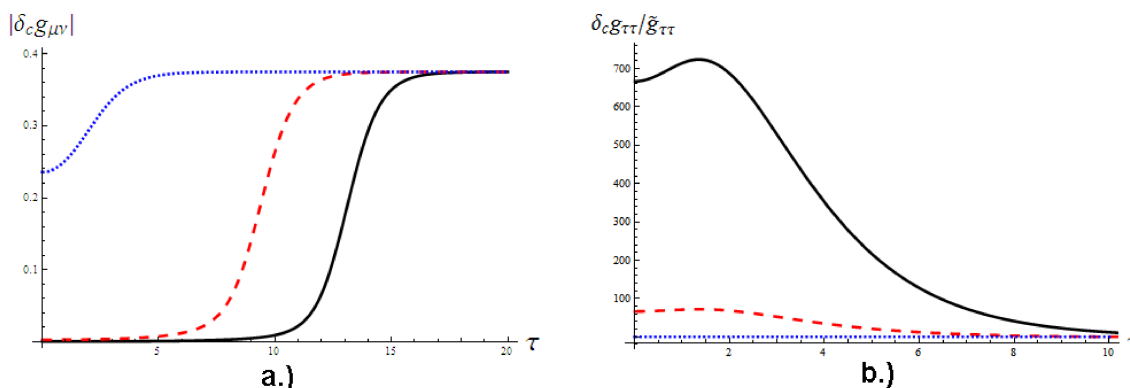
This solution is also the first time-dependent 10d background that correctly captures the nonlinear physics of modulus motion in warped string compactifications. Since it is precisely consistent with the expected effective field theory, it should end concerns raised in [23, 24] about the validity of the 4d effective theory.

Finally, it seems that this solution is likely to share a number of features with cosmological backgrounds in these compactifications; in particular, if the Kähler modulus is stabilized with a mass well below the warped KK scale, its motion will be well approximated by classical solutions. Developing cosmological backgrounds would be of relevance to models of inflation in string theory and could shed light on higher-dimensional or string physics in cosmology. Unfortunately, solving for the motion of the Kähler modulus in a cosmological background is already difficult at the 4d level, so we leave this issue as an open question.

## 7. Strongly warped limit and light KK modes

In the previous sections we have obtained the 10d solution corresponding to the universal Kähler modulus, first in the linearized approximation, and then showing how to include finite fluctuations. We also studied the 4d properties of the solution, by finding the Kähler potential and proving no-scale structure. In this section we will show how to apply our results to strongly warped throats in the compactification manifold.

Strongly warped regions are important both from a phenomenological point of view and to understand gauge/gravity dualities in string theory. Moreover, the effects from compensating fields are expected to dominate in this limit [1], so this is good place to illustrate our results. Another important dynamical effect is that at strong warping the KK mass scale is redshifted, and could become of the same order as the energy scale of the EFT for the moduli fields. Therefore, these new light fields need to be included in the 4d description. In the first part of the section we will find the 10d wavefunction of the volume modulus at strong warping, and illustrate its behavior for various choices of warping. Next we show will how to include light KK modes, concluding that there are no kinetic mixings with the Kähler modulus.



**Figure 1:** (a) The 4-dimensional wavefunction  $\delta_c g_{\mu\nu}$  and (b) the internal metric wavefunction  $\delta_c g_{\tau\tau} / \tilde{g}_{\tau\tau}$  in a Klebanov-Strassler warped background for various values of the warping evaluated at the tip  $e^{-4A_0(0)}$ : no warping  $e^{-4A_0(0)} = 1$ , dotted blue; weak warping  $e^{-4A_0(0)} = 10^4$ , dashed red; strong warping  $e^{-4A_0(0)} = 10^6$ , solid black. Notice that as the warping increases, the wavefunction dips deeper into the throat.

### 7.1 Wavefunction in the strongly warped limit

To begin with a simple example, consider an AdS warp factor  $e^{-4A_0} \sim N/r^4$ . Without including compensating fields, the 10d wavefunction corresponding to the volume modulus  $c(x)$  scales, at small  $r$ , like

$$\delta_c g_{\mu\nu} \sim \frac{r^6}{N^{3/2}}, \quad \delta_c g_{rr} \sim \frac{r^2}{N^{1/2}}. \quad (7.1)$$

On the other hand, including the effect of compensating fields, we obtain the qualitatively different behavior

$$\delta_c g_{\mu\nu} \sim \frac{r^2}{N^{1/2}}, \quad \delta_c g_{rr} \sim \frac{N^{1/2}}{r^2}. \quad (7.2)$$

This illustrates the point that the correct gauge invariant 10d fluctuation may differ significantly from the naive solution.

Let us be more concrete and model the throat locally by a warped deformed conifold with metric given by the the Klebanov-Strassler solution [34],

$$ds^2 = e^{2A_0} \eta_{\mu\nu} + e^{-2A_0} \frac{\epsilon^{4/3}}{2} K(\tau) \left[ \frac{d\tau^2 + (g^5)^2}{3K^3(\tau)} + \cosh^2\left(\frac{\tau}{2}\right) ((g^3)^2 + (g^4)^2) + \sinh^2\left(\frac{\tau}{2}\right) ((g^1)^2 + (g^2)^2) \right] \quad (7.3)$$

where  $\tau$  is the radial coordinate along the throat. The equation for the compensator (3.10) now becomes

$$\partial_\tau \left( K^2(\tau) \cosh^2 \frac{\tau}{2} \sinh^2 \frac{\tau}{2} B_\tau(\tau) \right) = \left( \frac{V_W^0}{V_{CY}} - e^{-4A_0(y)} \right) \frac{\epsilon^{4/3}}{6} \cosh^2 \frac{\tau}{2} \sinh^2 \frac{\tau}{2} \quad (7.4)$$

One can now solve this equation numerically for various values of the warping — the results for the wavefunctions  $\delta_c g_{\mu\nu}, \delta_c g_{\tau\tau} / \tilde{g}_{\tau\tau}$  are shown in figure 1. For convenience of

display in figure 1 we have divided out the unwarped part  $\tilde{g}_{\tau\tau}$  of the metric to show that at large  $\tau$ , where the warping is weak, the physical metric fluctuation asymptotes to the unfluctuated and unwarped metric, which is what we expect.

As the amount of warping increases (dashed red and solid black lines) the internal metric wavefunctions  $\delta_c g_{ij}$  become more peaked in the tip region of the throat where the warping is strongest, while the 4d metric wavefunctions  $\delta_c g_{\mu\nu}$  decrease to zero, as expected from our simple estimates with the AdS warp factor (7.2).

## 7.2 Inclusion of KK modes

We now address the problem of including light KK modes in the EFT of the volume modulus.<sup>8</sup> A general argument for the absence of kinetic mixings between zero modes and their KK excitations was given in [2]. It was based on the observation that these fluctuations are eigenvectors of a Sturm-Liouville problem, such that the orthogonality relation derived from the differential problem coincides with the Hamiltonian inner product. This then grants the absence of kinetic mixings. Since the application to  $p$ -forms may be unfamiliar, we now show that the universal axion is orthogonal to its KK excitations.

Consider then the 2-form massless and massive modes in  $C_4$ ,

$$\delta C_4 = a_2(x) \wedge \tilde{J}(y) + \sum_{\alpha} a_2^{\alpha}(x) \wedge \omega_{\alpha}(y) \tag{7.5}$$

where  $\omega_{\alpha}$  are (non-closed) 2-forms, and the KK fields  $a_2^{\alpha}$  are dual to spacetime scalars. The compensating fields are already absorbed into  $\tilde{J}$  and  $\omega_{\alpha}$ . For simplicity, we are also setting the Weyl factor equal to one. There are, of course, other components, and we have not determined the complete wavefunctions for the excited KK modes, but we can see orthogonality just from these components.

Requiring that the particles have a well-defined 4d mass,  $d(\hat{\star}_4 da_2^{\alpha}) = -m_{\alpha}^2 \hat{\star}_4 a_2^{\alpha}$ , we derive the eigenvector equation

$$d(\tilde{\star}_6 d\omega_{\alpha}) = m_{\alpha}^2 e^{-4A} \tilde{\star}_6 \omega_{\alpha}. \tag{7.6}$$

The computation of the kinetic mixing between  $a_2(x)$  and  $a_2^{\alpha}(x)$  then proceeds as in eq. (5.5):

$$\begin{aligned} \int E_5 \wedge \star_{10} E_5 &\rightarrow - \int_x a_2(x) \wedge d[\hat{\star}_4 da_2^{\alpha}(x)] \int_y e^{-4A(y)} \tilde{J} \wedge \tilde{\star}_6 \omega_{\alpha} \\ &= - \frac{1}{m_{\alpha}^2} \int_x a_2(x) \wedge d[\hat{\star}_4 da_2^{\alpha}(x)] \int_y \tilde{J} \wedge d(\tilde{\star}_6 d\omega_{\alpha}) \end{aligned} \tag{7.7}$$

where we have used (7.6). Since  $\tilde{J}$  is closed, integrating by parts the kinetic mixing vanishes.

By supersymmetry, the same holds for the universal volume modulus (since the analysis should not depend on our choice of 3-form flux, this statement holds even in classically nonsupersymmetric compactifications). We conclude that light KK modes do not mix with the Kähler modulus at the level of the kinetic terms.

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<sup>8</sup>We thank E. Silverstein for suggesting to check this.



## 8. Discussion and implications

By using the Hamiltonian method, developed for warped compactifications in [1], we have computed the kinetic term and Kähler potential for the universal volume modulus and its axionic partner in IIB flux compactifications of the type studied in [7] for arbitrary warping. We found that the Kähler potential for the universal Kähler modulus takes the form

$$K(\rho, \bar{\rho}) = -3 \log \left( -i(\rho - \bar{\rho}) - 2 \frac{V_W^0}{V_{CY}} \right). \quad (8.1)$$

It is rather striking that all warping corrections just amount to an additive shift  $\rho \rightarrow \rho - i(V_W/V_{CY})$ . One way to understand this result is to argue that the no-scale symmetry survives in the correct 10d warped solution. This protects the Kähler potential from further warping corrections.

It is important to emphasize that the 10d time-dependent solution that we have found is very different from the unwarped fluctuation. Therefore, the respective 4d theories are expected to be different as well, even if the Kähler potentials have the same functional dependence. In particular, once nonperturbative corrections of the form  $W = A e^{ia\rho}$  are included, the previous seemingly innocuous shift in  $\rho$  may produce qualitative changes in the field theory. This could become important in KKLT type models [25] that rely on the existence of a strongly warped region. It would be interesting to compute the prefactor  $A$  (see [35–37]) in strongly warped backgrounds, and see how our 10d solution modifies the discussion.

In section 7 we showed that the warped 10d fluctuations for a time-dependent universal volume modulus are peaked at the tip of the throat, and that there are no Kähler potential mixings with light KK modes. This can be relevant for phenomenological applications in which the coupling of the universal Kähler modulus to brane and bulk fields, obtained by the 10d wavefunction overlap, is important. Also, studying further the wavefunctions of the KK modes of the universal axion could shed light on the possibility of mixing through mass terms as well as be important for studying the behavior of perturbations in strongly warped throats.

We have also shown in section 6 that the 10d metric fluctuations can be promoted to a fully time-dependent, warped, 10d metric for the universal volume modulus by taking into account the backreaction on the 4d space. This is a first step towards finding cosmological solutions for time-dependent Kähler moduli, which may be relevant for models of inflation.

There are several future directions of interest. First, it is highly desirable to determine the Kähler potential for general Kähler moduli, which are not stabilized by 3-form flux on a generic CY. Another interesting related open problem is calculating the Kähler potential for modes that are stabilized by the 3-form flux; as discussed in [1, 10, 38, 2], the flux also modifies the 10d wavefunction in this case. On a slightly different tack, it is natural to extend our results to excited KK modes of the volume modulus and axion, along the lines of [2]. Finally, generalization of our nonlinear solution to cosmological backgrounds is an important problem for future work in string cosmology.

## Acknowledgments

We would like to thank C. Burgess, F. Denef, S. Franco, T. Grimm, S. Kachru, A. Maharana, A. Nacif, J. Polchinski, G. Shiu, E. Silverstein and A. Tomasiello for useful discussions and comments. M.R.D. and G.T. are supported in part by DOE grant DE-FG02-96ER40959. G.T. would like to thank KITP for their hospitality and support through the National Science Foundation Grant PHY05-51164. The work of A. R. F. is supported by the NSERC Discovery Grant program. B. U. is supported by NSERC and by fellowships from the Institute for Particle Physics (Canada) and Lorne Trottier (McGill).

## A. Gauge transformations and field redefinitions of $C_4$

The dimensional reduction of fluctuations of  $C_4$  in 3-form flux background is slightly subtle due to its nonstandard gauge transformations. We follow the discussion of [9], which considered the case of a torus orientifold in some detail.

In terms of the 4-form that couples electrically to a D3-brane

$$S_{\text{WZ}} = \mu_3 \int C_4, \tag{A.1}$$

the 5-form field strength is  $\tilde{F}_5 = dC_4 - C_2 H_3$ . The gauge transformations that leave  $\tilde{F}_5$  invariant are

$$C_4 \rightarrow C_4 + d\chi_3 + \zeta_1^C \wedge H_3, \quad C_2 \rightarrow C_2 + d\zeta_1^C, \quad B_2 \rightarrow B_2 + d\zeta_1^B. \tag{A.2}$$

In a background of nontrivial 3-form flux, the potentials  $B_2$  and  $C_2$  are well-defined only on coordinate patches, which must be glued together with gauge identifications  $\zeta^{B,C}$ . With a fixed choice of background potentials  $C_4$ ,  $B_2$ , and  $C_2$ , the gauge transformations  $\zeta^{B,C}$  are also fixed, so fluctuations  $\delta B_2$ ,  $\delta C_2$  must be globally defined on the internal manifold (on a torus, this means they are periodic). Hence, they have the appropriate behavior for dimensional reduction without any issue of gluing coordinate patches together.

The 4-form is slightly more complicated; the background  $C_4$  is also defined only on patches and glued together by the gauge transformation (A.2) with  $H_3$  the background flux. This means that the fluctuation also has a nontrivial gauge gluing  $\delta C_4 \rightarrow \delta C_4 + d\chi + \zeta^C \delta H_3$ . To simplify the gluing conditions, we can define  $\delta C'_4 = \delta C_4 - C_2 \delta B_2$  (to linear order); this is glued together by gauge transformations  $\delta C'_4 \rightarrow \delta C'_4 + d\chi'$  with  $\chi' = \chi - \lambda^C \delta B_2$ , which are trivial as long as there is no quantized 5-form flux. Therefore, the 4-form potential that follows ordinary dimensional reduction is  $\delta C'_4$ . The field strength and complete gauge transformations work out to be

$$\delta \tilde{F}_5 = d\delta C'_4 + \frac{ig_s}{2} (\delta A_2 \wedge \bar{G}_3 - \delta \bar{A}_2 \wedge G_3) \tag{A.3}$$

$$\delta C'_4 \rightarrow \delta C'_4 + \delta\chi' + \frac{ig_s}{2} (\bar{\zeta}^A \wedge G_3 - \zeta^A \wedge \bar{G}_3) \tag{A.4}$$

in terms of the complex potential  $A_2 = C_2 - \tau B_2$ ,  $G_3 = dA_2$ . Henceforth, we drop the prime on  $\delta C_4$ .

Lest this seem like a technical but nonphysical point, let us make two comments. First, this field redefinition allows us to define the fluctuation in the 5-form without reference to the background 2-form potentials, which is an immense simplification. Second, the redefined 4-form fluctuation does not couple directly to the D3-brane as in eq. (A.1). The field redefinition modifies the coupling of the 2-form fluctuations to the D3-branes.

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